BREAKAWAY OF A BUBBLE FROM A HEATING SURFACE

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In this paper we solve the problem of determining the breakaway diameter of the vapor bubble in the case of a large number of active vapor-forming centers. The motion of the liquid is discussed. The dynamic pressure of the liquid surrounding the vapor bubble and the radius of the bubble on breakaway from the heating surface are determined.

Pressure increase during the nucleate boiling of a liquid leads to a rapid increase in the number of active vapor-forming centers. The rate of change of the number of centers greatly exceeds the rate of reduction of the bubble breakaway diameter given by the Fritz formula

$$D_0 = 0.02\Theta_0 \sqrt{\frac{\sigma}{\gamma - \gamma''}}\,. \qquad (1)$$

In view of this the average area of the cell occupied by one vapor-forming center is reduced so much that the dimensions of this cell become less than the breakaway diameter given by formula (1). This does not lead to fusion of the bubbles growing on the heating surface, however, and there is a much greater reduction of the breakaway diameter D_0 than formula (1) indicates. This fact was noted in [1]. This effect can be explained if it is assumed that the individual growing vapor bubbles affect one another.

The aim of the present investigation was to determine the breakaway diameter of the bubble when there are a large number of vapor-forming centers. This means that in the determination of the breakaway diameter we must take into account the size of the cell occupied by one center. For the solution of this problem we consider both the static and dynamic pressure of the liquid on the bubble surface as forces acting on the bubble upon breakaway from the heating surface. The dynamic pressure of the liquid displaced by the bubble during its growth can be determined from the equations of liquid motion. Thus, to solve the problem we have to solve the system of motion equations with account for the continuity equation for the liquid flowing around the vapor bubble.

To simplify the problem we make several assumptions.

1. We assume that there are two schemes of bubble formation on the heating surface: a) growth and breakaway of all the bubbles occur simultaneously ($F = 1/n$); b) neighboring centers act at different times (in the case of square cells). In the latter case the cross-sectional area of the cell in which the bubble grows is doubled ($F = 2/n$) owing to the neighboring "nonactive "vapor-forming centers. This (the action of centers at different times) has been observed experimentally [2].

2. We neglect friction forces in the liquid and regard the liquid flow as potential. This simplification has no significant effect on the dynamics of the liquid close to the bubble, since friction forces are very insignificant in the flow of a liquid round a bubble.

3. We assume that the bubble remains spherical during its growth (until breakaway).

4. In the solution of the problem we assume the cell to be of circular section and neglect the circular velocities of the liquid (and their derivatives).

5. To simplify the problem we take the contact angle Θ_0 to be 90° (see the figure), which allows us to neglect the velocity of the bubble center.

In light of the adopted assumptions the continuity equation of the liquid for one cell can be written as follows (in spherical coordinates):

$$\frac{\partial}{\partial R}\left(R^2\frac{\partial\varphi}{\partial R}\right) + \frac{1}{\sin\Theta}\frac{\partial}{\partial\Theta}\left(\frac{\partial\varphi}{\partial\Theta}\sin\Theta\right) = 0 \qquad (2)$$

with the boundary conditions

$$v_R = v \quad \text{for} \quad R = R_1, \qquad (3)$$

$$v_\Theta = 0 \quad \text{for} \quad \Theta = \Theta_0, \qquad (4)$$

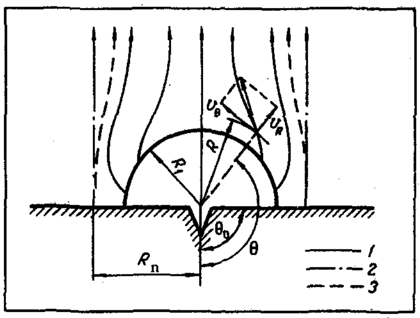

Diagram of motion of liquid displaced by growing bubble: 1) liquid flow lines; 2) boundary of cell; 3) arbitrary boundary of cell for adopted boundary condition (6).

$$v_R \sin\Theta + v_\Theta \cos\Theta = 0 \quad \text{for} \quad R\sin\Theta = R_n. \tag{5}$$

The general solution of Eq. (2) is a series in which the coefficients are determined from the boundary conditions [3]. Despite the adopted assumptions and simplifications, however, this solution is very complex. The main difficulty is the need to satisfy boundary condition (5). The solution of Eq. (2) can be greatly simplified if condition (5) is only partially satisfied (for instance, with $\Theta = \Theta_0 = 90°$). In this case

$$v_R = 0 \quad \text{for} \quad R = R_n \text{ and } \Theta = 90°. \tag{6}$$

The solution of Eq. (2) takes the form

$$\varphi = -vR_1\left\{\frac{R_1}{R} - \frac{R_n^2R_1^3}{R_n^5 - R_1^5}\left[\frac{1}{2}\left(\frac{R}{R_1}\right)^2 + \frac{1}{3}\left(\frac{R_1}{R}\right)^3\right](3\cos^2\Theta - 1)\right\}. \tag{7}$$

Condition (6) definitely introduces additional error into the general solution, particularly when $R \gg R_1$. We are interested, however, in the dynamic pressure on the bubble surface, i.e., at $R = R_1$. In this case the error is greatly reduced.

Knowing the velocity potential φ we can determine the projections of the liquid velocities in an individual cell,

$$v_R = \frac{\partial\varphi}{\partial R} = v\left\{\left(\frac{R_1}{R}\right)^2 + \frac{R_n^2R_1^3}{R_n^5 - R_1^5} \times \left[\frac{R}{R_1} - \left(\frac{R_1}{R}\right)^4\right](3\cos^2\Theta - 1)\right\}, \tag{8}$$

$$v_\Theta = \frac{1}{R}\frac{\partial\varphi}{\partial\Theta} = -v\frac{R_n^2R_1^3}{R_n^5 - R_1^5} \times \left[3\frac{R}{R_1} + 2\left(\frac{R_1}{R}\right)^4\right]\cos\Theta\sin\Theta. \tag{9}$$

The rate of growth of the bubble on the heating surface can be determined from the formula

$$v = \frac{dR_1}{d\tau} = \beta_*\frac{\lambda\Delta t}{r\rho'' R_1}, \tag{10}$$

which is derived from the condition for supply of heat from the heating surface to the base of the bubble [4] ($\beta_* = 6$).

To determine the pressure at the surface of the vapor bubble we use the equations of motion of an ideal liquid. Integrating these equations and taking into account that for an ideal liquid

$$\frac{\partial v_\Theta}{\partial\Theta} = \frac{\partial}{\partial R}(Rv_\Theta),$$

and using (10), we obtain the expression

$$\frac{p_d}{\rho} + \frac{v_R^2 + v_\Theta^2}{2} + C = -\int\frac{\partial v_R}{\partial\tau}dR = -\int\frac{\partial v_\Theta}{\partial\tau}Rd\Theta = v^2\left\{\frac{R_1}{R} - \frac{R_n^2R_1^3}{R_n^5 - R_1^5}\left[\frac{1}{2}\frac{R_n^5 + 4R_1^5}{R_n^5 - R_1^5}\left(\frac{R}{R_1}\right)^2 + \frac{1}{3}\frac{6R_n^5 - R_1^5}{R_n^5 - R_1^5}\left(\frac{R_1}{R}\right)^3\right](3\cos^2\Theta - 1)\right\}, \tag{11}$$

where C is the constant of integration.

Near the surface of the growing bubble, at $R = R_1$, Eq. (11), in view of (8) and (9), takes the form

$$\frac{p_d}{\rho} + C = \frac{v^2}{2}\left[1 - \left(5\frac{R_n^2R_1^3}{R_n^5 - R_1^5}\cos\Theta\sin\Theta\right)^2 - 5\frac{R_n^2R_1^3(3R_n^5 + 2R_1^5)}{(R_n^5 - R_1^5)^2}\left(\cos^2\Theta - \frac{1}{3}\right)\right]. \tag{12}$$

The constant of integration can be obtained from the condition

$$p_d = p_{d_0} \quad \text{for} \quad \Theta = \Theta_0 = 90°.$$

In this case

$$C = -\frac{p_{d_0}}{\rho} + \frac{v^2}{2}\left[1 + \frac{5}{3}\frac{R_n^2R_1^3(3R_n^5 + 2R_1^5)}{(R_n^5 - R_1^5)^2}\right]. \tag{13}$$

Substituting (13) into (12) we obtain the distribution of dynamic pressure of the liquid on the bubble surface

$$p_d = p_{d_0} - \rho\frac{v^2}{2}\left(\frac{3R_n^5 + 2R_1^5}{5R_n^2R_1^3} + \sin^2\Theta\right)\left(\frac{5R_n^2R_1^3}{R_n^5 - R_1^5}\right)^2\cos^2\Theta. \tag{14}$$

If the distribution of liquid static pressure over the bubble surface is written in a similar way we obtain the expression

$$p_c = p_{c_0} + \gamma R_1\cos\Theta. \tag{15}$$

According to the adopted notation (see Fig. 1), the angle Θ lies in the range $\Theta_0 < \Theta < \pi$. When $\Theta_0 = 90°$ the angle Θ lies between $\pi/2$ and π and, accordingly, $0 > \cos\Theta > -1$, i.e., $\cos\Theta$ is negative in the whole range of variation of angle Θ.

Thus, as Eq. (15) shows, with increase in the angle Θ the static pressure of the liquid decreases. A similar situation (reduction in dynamic pressure of liquid with increase in angle Θ) is revealed by Eq. (14). Hence, during the growth of the bubble on the heating surface the dynamic pressure has the same effect on the bubble as the static pressure, i.e., it makes the bubble break away from the heating surface.

The pressure of the liquid on the bubble surface is balanced by the vapor pressure inside the bubble. The nonuniform distribution of pressure over the bubble surface is compensated for by surface-tension forces. The condition for equality of pressures with the surface tension taken into account is written in the form

$$p'' - p = \sigma\left(\frac{1}{R_1'} + \frac{1}{R_1''}\right), \tag{16}$$

where R_1' and R_1'' are the main radii of curvature of the phase interface at the given point.

To determine the breakaway force P_1 acting on the bubble owing to the nonuniform pressure we take the integral of the pressure over the bubble surface

$$P_1 = \int_{\Theta_0}^{\pi}\Delta p\, 2\pi R_1^2\cos\Theta\sin\Theta\, d\Theta. \tag{17}$$

Here Δp is the total liquid pressure drop (static and dynamic) over the bubble surface with allowance for the static vapor pressure inside the bubble:

$$\Delta p = (p_s - p_{s0}) - (p''_s - p''_{s_0}) + (p_d - p_{d_0}) =$$
$$= (\gamma - \gamma'') R_1 \cos\Theta - \frac{\rho v^2}{2}\left(\frac{3R_n^5 + 2R_1^5}{5R_n^2 R_1^3} + \right.$$
$$\left. + \sin^2\Theta\right)\left(\frac{5R_n^2 R_1^3}{R_n^5 - R_1^5}\right)^2 \cos^2\Theta. \qquad (18)$$

Substituting (18) into (17) and integrating (with $\Theta_0 = 90°$), we obtain

$$P_1 = \frac{2}{3}\pi R_1^3 (\gamma - \gamma'') +$$
$$+ \pi R_1^2 \frac{\rho v^2}{2}\frac{5}{6}\frac{R_n^2 R_1^3 (9R_n^5 + 6R_1^5 + 5R_n^2 R_1^3)}{(R_n^5 - R_1^5)^2}. \qquad (19)$$

The breakaway force acting on the bubble is balanced by the surface-tension force, which holds the bubble on the heating surface. This force is given by the expression

$$P_2 = 2\pi R_1 \sigma f(\Theta_0). \qquad (20)$$

When the bubble attains the breakaway diameter the two forces acting on the bubble become equal, i. e.,

$$P_1 = P_2 = P_0 \quad \text{for} \quad R_1 = R_0.$$

Thus, the condition for breakaway of the bubble from the heating surface can be written in the form of the equation

$$2\pi R_0 \sigma f(90°) =$$
$$= \frac{2}{3}\pi R_0^3 (\gamma - \gamma'') + \pi R_0^2 \frac{\rho v_0^2}{2} 7{,}5F\left(\frac{R_0}{R_n}\right), \qquad (21)$$

where

$$F\left(\frac{R_0}{R_n}\right) = \left(\frac{R_0}{R_n}\right)^3 \frac{1 + \frac{5}{9}\left(\frac{R_0}{R_n}\right)^3 + \frac{2}{3}\left(\frac{R_0}{R_n}\right)^5}{\left[1 - \left(\frac{R_0}{R_n}\right)^5\right]^2}.$$

For simplification the function $F(R_0/R_n)$ can be approximated (to within 5%) by the relationship

$$F\left(\frac{R_0}{R_n}\right) = \left(\frac{R_0}{R_n}\right)^3 \frac{1}{1 - \left(\frac{R_0}{R_n}\right)^2}. \qquad (22)$$

If the cell is taken as approximately circular the mean radius of the cell (over the heating surface) will be

$$R_n = \frac{1}{\sqrt{\pi n}}. \qquad (23)$$

In determining the dynamic pressure of the liquid we ignored the effect of the contact angle Θ_0 and took its value as 90°. To take into account the contact angle we will assume in a first approximation that its effect is similar to the effect of Θ_0 on the breakaway of the bubble due to static pressure (see formula (1)).

In this case, by solving Eq. (21) using (10), (22), and (23) for R_0, we obtain

$$D_0 = 2R_0 = \sqrt{\frac{8}{\pi n}} \frac{1}{\sqrt{M + N + \sqrt{(M+N)^2 - 4M}}}, \qquad (24)$$

where

$$M = \left(\frac{100}{\Theta_0}\right)^2 \frac{\gamma - \gamma''}{\pi n \sigma};$$
$$N = 1 + 120\left(\frac{100}{\Theta_0}\right)^2 \sqrt{n}\,\frac{\rho}{\sigma}\left(\frac{\lambda \Delta t}{r \rho''}\right)^2.$$

Formula (24) gives the relationship between the breakaway diameter of the bubble and the physical properties of the heat-carrier, the temperature difference, and the number of vapor-forming centers.

Expressions M and N characterize the static and dynamic components, respectively, of the liquid pressure on the vapor bubble.

When the number of vapor-forming centers is small ($M > 1$), $N \to 1$ and formula (24) becomes (1). In this case the main effect on the bubble is due to the dynamic pressure (D_0 is a function of n).

Thus, formula (24) is a more general expression (in comparison with (1)) for calculation of the bubble breakaway diameter and can be extended to the region of higher saturation pressures.

To determine the number of vapor-forming centers in the calculation of D_0 (when more accurate data are lacking) we can use the formula given in [5]:

$$n = l\left(\frac{r \rho'' \Delta t}{T_s \sigma}\right)^3, \qquad (25)$$

where l is a coefficient with the dimension of length, $l = 625 \cdot 10^{-16}$ m.

NOTATION

R and Θ are spherical coordinates; R_1 is the radius of vapor bubble growing on heating surface; R_0 is the bubble radius at breakaway; R_n is the dimension of cell (radius of round cell); Θ_0 is the angle of contact with heating surface; τ is the time; φ is the velocity potential; v_R, v_Θ are the projections of velocities on coordinate axes in spherical system of coordinates; v is the rate of growth of vapor bubble on heating surface; v_0 is the rate of growth of bubble at moment of breakaway from heating surface; p_s and p_d are the static and dynamic components, respectively, of liquid pressure on bubble surface; p_{s_0} and p_{d_0} are the components of liquid pressure at base of vapor bubble ($\Theta = \Theta_0$); p''_s and p''_{s_0} are the static vapor pressure inside bubble; Δt is the wall-liquid temperature difference; n is the number of vapor-forming centers on surface; ρ'' is the vapor density; ρ is the liquid density; γ'' is the specific gravity of vapor; r is the latent heat of evaporation; σ is the surface tension; λ is the thermal conductivity of liquid; D_0 is the bubble diameter at breakaway.

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